

The role of $q\bar{q}$ components in the $N(1440)$ resonance

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Abstract

The role of 5-quark components in the pion and electromagnetic decays and transition form factors of the $N(1440)$ resonance is explored. The $qqqq\bar{q}$ components, where the 4-quark subsystem has the flavor-spin symmetries $[4]_{FS}[22]_F[22]_S$ and $[4]_{FS}[31]_F[31]_S$, which are expected to have the lowest energy of all $qqqq\bar{q}$ configurations, are considered in detail with a nonrelativistic quark model. The matrix elements between the 5-quark components of the $N(1440)$ and the nucleon, $qqqq\bar{q} \rightarrow qqqq\bar{q}$, play a minor role in these decays, while the transition matrix elements $qqqq\bar{q} \rightarrow qqq$ and $qqq \rightarrow qqqq\bar{q}$ that involve quark antiquark annihilation are very significant. Both for electromagnetic and strong decay the change from the valence quark model value is dominated by the confinement triggered $q\bar{q}$ annihilation transitions. In the case of pion decay the calculated decay width is enhanced substantially both by the direct $q\bar{q} \rightarrow \pi$ and also by the confinement triggered $q\bar{q} \rightarrow \pi$ transitions. Agreement with the empirical value for the pion decay width may be reached with a $\sim 30\%$ $qqqq\bar{q}$ component in the $N(1440)$.

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I. INTRODUCTION

The conventional 3-quark model for nucleons and nucleon resonances leads to values for both the electromagnetic and the strong decay widths of the lowest nucleon resonance, ($N(1440)$), which are smaller by an order of magnitude than the corresponding experimental values [1, 2]. This suggests that this resonance should have significant $qq\bar{q}$, if not more exotic components, in addition to the 3 valence quarks. This is also suggested by the fact that this resonance appears naturally at its very low energy as a vibrational - i.e collective - state in the Skyrme model [3]. This issue is studied here by explicit consideration of the $qqqq\bar{q}$ configurations, which are expected to have the lowest energy, and which therefore may be expected to form a significant components in this resonance.

Admixture of a $qqqq\bar{q}$ component with a probability of $\sim 30\%$ is found to increase the calculated helicity amplitude $A_{\frac{1}{2}}$ for $N(1440) \rightarrow N\gamma$ decay moderately and the width of $N(1440) \rightarrow N\pi$ decay significantly. While the matrix elements between the 5-quark components of the $N(1440)$ and the nucleon, $qqqq\bar{q} \rightarrow qqqq\bar{q}$, play but a minor role in these decays, the transition matrix elements $qqqq\bar{q} \rightarrow qqq$ and $qqq \rightarrow qqqq\bar{q}$ that involve quark antiquark annihilation are very significant. The calculated helicity amplitude and the modification from the valence quark model value is very sensitive to the transitions between the $qqqq\bar{q}$ and qqq components that are triggered by the confinement between quarks. In the case of pion decay the calculated decay width is substantially enhanced by the direct transition and also by the confinement triggered transitions between the $qqqq\bar{q}$ and qqq components. It is found that with a $\sim 30\%$ $qqqq\bar{q}$ component in the $N(1440)$ it becomes possible to reach agreement with the empirical pion decay width. It is also found that recent data on the shape of the empirical helicity amplitude $A_{\frac{1}{2}}$ for the $N(1440) \rightarrow N\gamma$ transition demand a spatially extended wave function model.

This paper is arranged as follows: The $qqqq\bar{q}$ wave functions of the proton and the $N(1440)$ are described in Section II. In section III the electromagnetic decay of the $N(1440)$ resonance is studied by explicit calculation of the effect of transitions between the $qqqq\bar{q}$ components of the proton and the $N(1440)^+$ and the corresponding annihilation transitions $qqqq\bar{q} \rightarrow qqq\gamma$ and $qqq \rightarrow qqqq\bar{q}\gamma$. In Section IV the strong decay of the $N(1440)^+$ is considered along the lines of the electromagnetic decay. Section IV contains a concluding discussion.

II. THE WAVE FUNCTIONS OF NUCLEON AND $N(1440)$

With addition of a $qqqq\bar{q}$ component in addition to the conventional qqq configuration, the complete wave functions of the nucleon and the $N(1440)$ resonance may be written schematically in the form:

$$\begin{aligned} |N, s_z\rangle &= A_{p3}|N, s_z\rangle_{3q} + A_{p5}|N, s_z\rangle_{5q}, \\ |N(1440), s_z\rangle &= A_{r3}|N(1440), s_z\rangle_{3q} + A_{r5}|N(1440), s_z\rangle_{5q}. \end{aligned} \quad (1)$$

Here A_{p3} and A_{p5} are the amplitude factors for the qqq and $qqqq\bar{q}$ components in the nucleon and A_{r3} and A_{r5} are the corresponding factors for the $N(1440)$, respectively. Their spin projection on the z -axis is denoted s_z . The matrix elements of the amplitudes for electromagnetic and strong decay will be formed of diagonal transitions terms: $qqq \rightarrow qqq$, $qqqq\bar{q} \rightarrow qqqq\bar{q}$ as well as of non-diagonal terms of the type $qqqq\bar{q} \rightarrow qqq$ and $qqq \rightarrow qqqq\bar{q}$, which involve quark-antiquark annihilation.

A. The wave functions of the qqq components

In the harmonic oscillator quark model the wave functions of the nucleon and the $N(1440)$ in the qqq configuration have the standard expressions:

$$\begin{aligned} |N, s_z\rangle_{3q} &= \frac{1}{\sqrt{2}} \left(|\frac{1}{2}, t_z\rangle_+ |\frac{1}{2}, s_z\rangle_+ + |\frac{1}{2}, t_z\rangle_- |\frac{1}{2}, s_z\rangle_- \right) \phi_{000}(\vec{\xi}_2) \phi_{000}(\vec{\xi}_1), \\ |N(1440), s_z\rangle_{3q} &= \frac{1}{\sqrt{2}} \left(|\frac{1}{2}, t_z\rangle_+ |\frac{1}{2}, s_z\rangle_+ + |\frac{1}{2}, t_z\rangle_- |\frac{1}{2}, s_z\rangle_- \right) \\ &\quad \cdot \frac{1}{\sqrt{2}} \left(\phi_{200}(\vec{\xi}_2) \phi_{000}(\vec{\xi}_1) + \phi_{000}(\vec{\xi}_2) \phi_{200}(\vec{\xi}_1) \right). \end{aligned} \quad (2)$$

Here $|\frac{1}{2}, s_z\rangle_\pm$ and $|\frac{1}{2}, t_z\rangle_\pm$, with t_z being the isospin- z component, are spin and isospin wave functions of mixed symmetry ([21]), in which (+) denotes a state that is symmetric “(112)” and (-) denotes a state that is antisymmetric “(121)” under exchange of the spin or isospin of the first two quarks. The Jacobi coordinates $\vec{\xi}_1$ and $\vec{\xi}_2$ are defined by the 3-quark position coordinates as:

$$\vec{\xi}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2), \quad \vec{\xi}_2 = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3). \quad (3)$$

The harmonic oscillator wave functions are:

$$\phi_{000}(\vec{\xi}_i) = \left(\frac{\omega_3^2}{\pi}\right)^{\frac{3}{4}} e^{-\omega_3^2 \xi_i^2/2}, \quad \phi_{200}(\vec{\xi}_i) = \sqrt{\frac{2}{3}} \omega_3^2 (\xi_i^2 - \frac{3}{2\omega_3^2}) \phi_{000}(\vec{\xi}_i). \quad (4)$$

Here the subscripts denote the quantum numbers (nlm) of the oscillator wave functions.

B. The wave functions of the $qqqq\bar{q}$ components

Positive parity demands that the $qqqq\bar{q}$ admixtures in the proton and the $N(1440)^+$ be P -wave states (or states with higher odd angular momentum). By the conventional assumption that the hyperfine interaction between the quarks is spin-dependent, it follows that the $qqqq$ configurations that have the lowest energy is that, for which the spin state has the highest possible degree of antisymmetry [4]. A similar argument applies in the case the hyperfine interaction is flavor dependent. The simplest hyperfine interaction model, which leads to a realistic splitting between the nucleon and the $N(1440)$ is the schematic flavor and spin dependent hyperfine interaction between the quarks $-C_\chi \sum_{i<j} \vec{\lambda}_F^i \cdot \vec{\lambda}_F^j \vec{\sigma}_i \cdot \vec{\sigma}_j$, where C_χ is a constant parameter ($C_\chi \sim 20-30$ MeV) [5]. This implies that the $qqqq\bar{q}$ configuration that has the lowest energy, and which is most likely to form notable admixtures in the nucleons and the $N(1440)$, has the mixed spin-flavor symmetry $[4]_{FS}[22]_F[22]_S$, with the antiquark in the ground state [4]. The flavor-spin symmetry of the state with the next to lowest energy is $[4]_{FS}[31]_F[31]_S$.

The $qqqq\bar{q}$ component in the proton with spin-flavor symmetry $[4]_{FS}[22]_F[22]_S$ has mixed spatial symmetry $[31]_X$, and may be represented by the wave function:

$$|N, s_z\rangle_{5q} = \frac{1}{\sqrt{2}} \sum_{a,b} \sum_{m,s} (1, 1/2, m, s | 1/2, s_z) C_{[31]_a[211]_a}^{[14]} [211]_C(a) [31]_{X,m}(a) [22]_F(b) [22]_S(b) \bar{\chi}_{t_z,s} \psi(\{\vec{\xi}_i\}). \quad (5)$$

Here the color, space and flavor-spin wave functions of the $qqqq$ subsystem have been denoted by their Young patterns respectively, and the sum over a runs over the 3 configurations of the $[211]_C$ and $[31]_X$ representations of S_4 , and the sum over b runs over the 2 configurations of the $[22]$ representation of S_4 respectively [6]. Here $C_{[31]_a[211]_a}^{[14]}$ denotes the S_4 Clebsch-Gordan coefficients of the representations $[1111][31][211]$.

The explicit color-space part of the wave function (5) may be expressed in the form:

$$\psi_C(\{\vec{\xi}_i\}) = \frac{1}{\sqrt{3}} \{ C_1 \varphi_{01m}(\vec{\xi}_1) \varphi_{000}(\vec{\xi}_2) \varphi_{000}(\vec{\xi}_3) \varphi_{000}(\vec{\xi}_4) - C_2 \varphi_{000}(\vec{\xi}_1) \varphi_{01m}(\vec{\xi}_2) \varphi_{000}(\vec{\xi}_3) \varphi_{000}(\vec{\xi}_4) + C_3 \varphi_{000}(\vec{\xi}_1) \varphi_{000}(\vec{\xi}_2) \varphi_{01m}(\vec{\xi}_3) \varphi_{000}(\vec{\xi}_4) \}. \quad (6)$$

Here the additional coordinate vectors $\vec{\xi}_i$, $i=3,4$, are defined as:

$$\begin{aligned}\vec{\xi}_3 &= \frac{1}{\sqrt{12}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4), \\ \vec{\xi}_4 &= \frac{1}{\sqrt{20}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5),\end{aligned}\quad (7)$$

and the oscillator wave functions are defined as:

$$\varphi_{000}(\vec{\xi}_i) = \left(\frac{\omega_5^2}{\pi}\right)^{\frac{3}{4}} e^{-\omega_5^2 \xi_i^2/2}, \quad \varphi_{01m}(\vec{\xi}_i) = \sqrt{2}\omega_5 \xi_{i,m} \varphi_{000}(\vec{\xi}_i). \quad (8)$$

Note that the oscillator parameter for the $qqqq\bar{q}$ component ω_5 may differ from that in the qqq component (4). In Eq. (6) C_i ($i=1, 2, 3$) represent the color wave functions of the 3 configurations of $[211]_C$ and notice has been taken that the vectors $\vec{\xi}_i$ ($i=1, 2, 3$) realize the 3 symmetry configurations of $[31]_X$ in orbital space [6].

In the present model, the wave function of the 5-quark component of $N(1440)$ has the same structure as the nucleon in the spin-flavor-color space. Orthogonality against the nucleon determines the corresponding wave function in the orbital space. Schematically the $N(1440)$ wave function in the $qqqq\bar{q}$ configuration with the lowest energy may be written as [4]:

$$\begin{aligned}|N(1440), s_z\rangle_{5q} &= \frac{1}{\sqrt{2}} \sum_{a,b} \sum_{m,s} (1, 1/2, m, s | 1/2, s_z) C_{[31]_a[211]_a}^{[1^4]} \\ &[211]_C(a) [31]_{X,m}(a) [22]_F(b) [22]_S(b) \bar{\chi}_{t_z,s} \Psi(\{\vec{\xi}_i\}).\end{aligned}\quad (9)$$

Here the orbital wave function of the $qqqq\bar{q}$ component $\Psi(\{\vec{\xi}_i\})$ lies in the $n = 2$ band and is combined with the color wave functions $[211]_C$ and the spatial wave functions $[31]_X$ in the following way:

$$\begin{aligned}\Psi_C(\{\vec{\xi}_i\}) &= \frac{1}{2\sqrt{3}} \{C_1 [\varphi_{21m}(\vec{\xi}_1)\varphi_{000}(\vec{\xi}_2)\varphi_{000}(\vec{\xi}_3)\varphi_{000}(\vec{\xi}_4) + \varphi_{01m}(\vec{\xi}_1)\varphi_{200}(\vec{\xi}_2)\varphi_{000}(\vec{\xi}_3)\varphi_{000}(\vec{\xi}_4) \\ &+ \varphi_{01m}(\vec{\xi}_1)\varphi_{000}(\vec{\xi}_2)\varphi_{200}(\vec{\xi}_3)\varphi_{000}(\vec{\xi}_4) + \varphi_{01m}(\vec{\xi}_1)\varphi_{000}(\vec{\xi}_2)\varphi_{000}(\vec{\xi}_3)\varphi_{200}(\vec{\xi}_4)] - \\ &C_2 [\varphi_{200}(\vec{\xi}_1)\varphi_{01m}(\vec{\xi}_2)\varphi_{000}(\vec{\xi}_3)\varphi_{000}(\vec{\xi}_4) + \varphi_{000}(\vec{\xi}_1)\varphi_{21m}(\vec{\xi}_2)\varphi_{000}(\vec{\xi}_3)\varphi_{000}(\vec{\xi}_4) \\ &+ \varphi_{000}(\vec{\xi}_1)\varphi_{01m}(\vec{\xi}_2)\varphi_{200}(\vec{\xi}_3)\varphi_{000}(\vec{\xi}_4) + \varphi_{000}(\vec{\xi}_1)\varphi_{01m}(\vec{\xi}_2)\varphi_{000}(\vec{\xi}_3)\varphi_{200}(\vec{\xi}_4)] + \\ &C_3 [\varphi_{200}(\vec{\xi}_1)\varphi_{000}(\vec{\xi}_2)\varphi_{01m}(\vec{\xi}_3)\varphi_{000}(\vec{\xi}_4) + \varphi_{000}(\vec{\xi}_1)\varphi_{200}(\vec{\xi}_2)\varphi_{01m}(\vec{\xi}_3)\varphi_{000}(\vec{\xi}_4) \\ &+ \varphi_{000}(\vec{\xi}_1)\varphi_{000}(\vec{\xi}_2)\varphi_{21m}(\vec{\xi}_3)\varphi_{000}(\vec{\xi}_4) + \varphi_{000}(\vec{\xi}_1)\varphi_{000}(\vec{\xi}_2)\varphi_{01m}(\vec{\xi}_3)\varphi_{200}(\vec{\xi}_4)]\}.\end{aligned}\quad (10)$$

The additional harmonic oscillator wave functions here are defined as:

$$\varphi_{200}(\vec{\xi}_i) = \sqrt{\frac{2}{3}}\omega_5^2(\xi_i^2 - \frac{3}{2\omega_5^2})\varphi_{000}(\vec{\xi}_i), \quad \varphi_{21m}(\vec{\xi}_i) = \frac{2}{\sqrt{7}}\omega_5^3 \xi_{i,m}(\xi_i^2 - \frac{3}{2\omega_5^2})\varphi_{000}(\vec{\xi}_i). \quad (11)$$

In the case of the flavor-spin symmetry configuration $[4]_{FS}[31]_F[31]_S$ the wave function of the $qqqq\bar{q}$ component of the nucleon takes the form:

$$\Psi_{[31]}^{(J)}(s_z) = \frac{1}{\sqrt{3}} \sum_{a,b} \sum_{m,s,M,j;T,t} (1, 1, m, M | J, j)(J, 1/2, j, s | 1/2, s_z) C_{[211]a, [31]a}^{[1^4]} (1, 1/2, T, t | 3/2, 1/2) [211]_C(a) [31]_{X,m}(a) [31]_{F,T}(b) [31]_{S,M}(b) \bar{\chi}_{t,s} \psi(\{\vec{\xi}_i\}). \quad (12)$$

Here J denotes the total angular momentum of the $qqqq$ system, which takes the values 0 and 1. The sum over a again runs over the 3 configurations of the $[211]_C$ and $[31]_X$ representations of S_4 . The sum over b runs over the 3 configurations of the $[31]$ representation. Here the isospin-z component of the 4-quark state is denoted T and that of the antiquark t . The corresponding wave function for the excited $qqqq\bar{q}$ component is obtained by replacement of the spatial wave function $\psi(\{\vec{\xi}_i\})$ by the corresponding wave function $\Psi(\{\vec{\xi}_i\})$ (9).

III. THE ELECTROMAGNETIC DECAY $N(1440)^+ \rightarrow p\gamma$

A. $qqq \rightarrow qqq\gamma$ matrix elements

Consider $N(1440)^+ \rightarrow p\gamma$ decay that is driven by the direct electromagnetic coupling to individual constituent quarks ($qq\gamma$). In the non-relativistic approximation the matrix elements of the elastic and annihilation matrix elements of the current operator are then:

$$\begin{aligned} \langle \vec{p}' | \vec{j} | \vec{p} \rangle_{elas} &= \frac{\vec{p}' + \vec{p}}{2m} + \frac{i}{2m} (\vec{\sigma} \times \vec{q}), \\ \langle \vec{p}' | \vec{j} | \vec{p} \rangle_{anni} &= \vec{\sigma}, \end{aligned} \quad (13)$$

respectively. For pointlike quarks the electromagnetic transition operator for elastic transitions between states with n_q quarks is then:

$$T = \sum_{i=1}^{n_q} \sqrt{2} \frac{e_i}{2m} \sigma_{i-} k_\gamma. \quad (14)$$

Here e_i and m are the electric charge and mass of the quark that emits the photon, respectively, and n_q is the number of constituent quarks. Here the momentum of the final right-handed virtual-photon is taken to be in the direction of the z -axis: $\vec{k} = (0, 0, k_\gamma)$ in the center of mass frame of the $N(1440)$. It is related to the magnitude of four-momentum transfer $Q = \sqrt{-k^2}$ as:

$$k_\gamma^2 = Q^2 + \frac{(M_R^2 - M_N^2 - Q^2)^2}{4M_R^2}, \quad (15)$$

where M_R and M_N are masses of the $N(1440)$ and the nucleon, respectively. In Eq. (14) the term that is proportional to quark momentum has been omitted as it does not contribute to the transition under consideration.

The $N(1440)^+ \rightarrow p\gamma$ transition is described by the helicity amplitude,

$$A_{\frac{1}{2}} = \frac{1}{\sqrt{2K_\gamma}} \langle p, \frac{1}{2}, -\frac{1}{2} | T | N(1440)^+, \frac{1}{2}, \frac{1}{2} \rangle, \quad (16)$$

for the helicity component $1/2$ of the $N(1440)$ on the direction of the photon momentum. Here $K_\gamma = 414$ MeV is the real-photon three momentum in the center of mass frame of $N(1440)$.

The helicity amplitude for $N(1440)^+ \rightarrow p\gamma$ decay in the conventional qqq configuration may be calculated from Eq. (2) as:

$$A_{\frac{1}{2}}^{(3q)} = -\frac{\sqrt{3}}{18} \frac{k_\gamma^2}{\omega_3^2} e^{-k_\gamma^2/6\omega_3^2} \frac{e}{2m} \frac{k_\gamma}{\sqrt{K_\gamma}}. \quad (17)$$

It is instructive to consider this expression as a function of the parameter values. It follows from the expression that the main sensitivity of the calculated helicity amplitude is to the oscillator frequency, which is inversely proportional to the spatial extent of the wave function. This is illustrated in Fig. 1, where the helicity amplitude (17) is shown as a function of Q^2 for two values of ω_3 when the constituent quark mass is taken to be $m = 340$ MeV. The parameter values are $\omega_3 = 110$ MeV and $\omega_3 = 225$ MeV, the latter set by the empirical radius of the proton as $\omega_3 = 1/r_p$. The data points in the figure are those given in ref. [12]. It is clear that the empirical shape of the helicity amplitude is much better recovered with the smaller value 110 MeV of ω_3 . This suggests that the $N(1440)$ has a much more extended structure than the nucleon. We shall employ this smaller value for ω_3 below. At the photon point, $Q^2 = 0$, $k_\gamma = K_\gamma$ and the expression Eq. (17) leads to the value $A_{\frac{1}{2}}^{(3q)} = -0.037/\sqrt{\text{GeV}}$, which is smaller than the corresponding experimental value $A_{\frac{1}{2}} = -0.065 \pm 0.004/\sqrt{\text{GeV}}$ by a factor 1.8 [7].

B. $qqqq\bar{q} \rightarrow qqqq\bar{q}\gamma$ transition matrix elements

The $qqqq\bar{q} \rightarrow qqqq\bar{q}\gamma$ transition matrix elements are the matrix elements of the transition operator (14) between the wave functions (5) and (9) that represent the $qqqq\bar{q}$ components of the nucleon and the $N(1440)$ resonance respectively. The mixed symmetry configuration

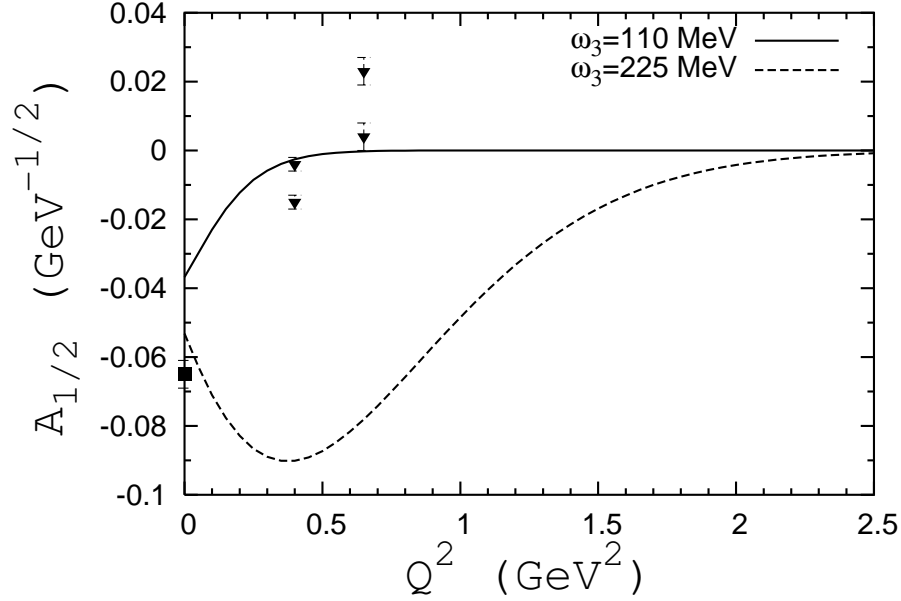


FIG. 1: The helicity amplitude for $N(1440)^+ \rightarrow p\gamma$ in the qqq model for 2 values of the oscillator frequency ω_3 . The data point at $Q^2 = 0$ is from [7] (square) and the other points are taken from the phenomenological analysis in [12] (triangles).

$[22]_S$ of the $qqqq$ subsystem has total spin 0, and as a consequence the matrix elements of the current operators of these quarks vanish and do not contribute to the direct transition $N(1440) \rightarrow N$. Therefore the $qqqqq\bar{q} \rightarrow qqqq\bar{q}\gamma$ transition amplitude only comes from the direct transitions between the antiquarks: $\bar{q} \rightarrow \bar{q}\gamma$, the transition operator of which has the opposite sign to that of $q \rightarrow q\gamma$.

For these, the matrix element of the spin-flavor-color part of the operator is found to be $C_{SFC}^{(5q)} = 1/9$. The matrix element in orbital space calculated with the wave functions given by Eqs. (6) and (10) becomes:

$$\langle \psi_C(\{\xi_i\}) | e^{-i\vec{k}_\gamma \cdot \vec{r}_5} | \Psi_C(\{\xi_i\}) \rangle = -\frac{\sqrt{6}}{30} \frac{k_\gamma^2}{\omega_5^2} e^{-k_\gamma^2/5\omega_5^2}. \quad (18)$$

The contribution from these diagonal $qqqqq\bar{q}$ matrix elements to the helicity amplitude for $N(1440)^+ \rightarrow p\gamma$ decay is then found to be:

$$A_{\frac{1}{2}}^{(5q)} = -\frac{\sqrt{6}}{270} \frac{k_\gamma^2}{\omega_5^2} e^{-k_\gamma^2/5\omega_5^2} \frac{e}{2m} \frac{k_\gamma}{\sqrt{K_\gamma}}. \quad (19)$$

If the value of the oscillator parameter of the $qqqq\bar{q}$ component is taken to be $\omega_5 = 200$ MeV the numerical value for this contribution $A_{\frac{1}{2}}^{(5q)}$ to the helicity amplitude is very small: $-0.0047/\sqrt{\text{GeV}}$ at the photon point $Q^2 = 0$. This value is much smaller than that given by the conventional qqq model. The reason for this small value is due to the fact that for the configuration with spin symmetry [22] the 4 quarks do not contribute to the decay amplitude, so that the whole contribution arises from the single antiquark. The result is not very sensitive to the value of ω_5 .

With the normalizations of the wave functions of nucleon and $N(1440)$ in (1), the diagonal contributions to the helicity amplitude for $N(1440)^+ \rightarrow p\gamma$ is obtained by addition of the $qqqq\bar{q} \rightarrow qqqq\bar{q}\gamma$ and the conventional $qqq \rightarrow qqq\gamma$ amplitudes:

$$A_{\frac{1}{2}} = A_{p3}A_{r3}A_{\frac{1}{2}}^{(3q)} + A_{p5}A_{r5}A_{\frac{1}{2}}^{(5q)}. \quad (20)$$

As the 5-quark helicity amplitude in (19) is much smaller than the conventional 3-quark helicity amplitude in (17), the combination of these two mechanisms result in a final helicity amplitude, which is in worse disagreement with the empirical value than the value given by the qqq model, even with assumption of a large component of $qqqq\bar{q}$ in the $N(1440)$ resonance. For instance, assuming a 10% proportion of the 5-quark component in proton and 30% in $N(1440)$, or equivalently the normalizations $A_{p3} = 0.95$ and $A_{r3} = 0.84$, leads to $A_{\frac{1}{2}} = -0.030/\sqrt{\text{GeV}}$ at $Q^2 = 0$ if the phase of the normalizations A_{p5} and A_{r5} are assumed to be +1, which is smaller than the empirical value by a factor ~ 2 .

C. Transitions through $q\bar{q}$ annihilation

1. Direct $qqqq\bar{q} \rightarrow qqq\gamma$ and $qqq \rightarrow qqqq\bar{q}\gamma$ transitions

In the case of $q\bar{q} \rightarrow \gamma$ transitions the γ_μ coupling for pointlike quarks (cf. (13)) leads to the following $q\bar{q} \rightarrow \gamma$ transition operator that involves annihilation of $q\bar{q}$ pair into a right handed photon (Fig. 2):

$$T_a = \sum_{i=1}^4 \sqrt{2} e_i \sigma_{i-}. \quad (21)$$

Here e_i is the electric charge of the annihilating quark and σ_{i-} is the spin lowering operator.

For the case of the $qqqq\bar{q} \rightarrow qqq\gamma$ transition in Fig.2(a), the calculation of the matrix element of the operator (21) involves calculation of the overlap of the qqq component of the

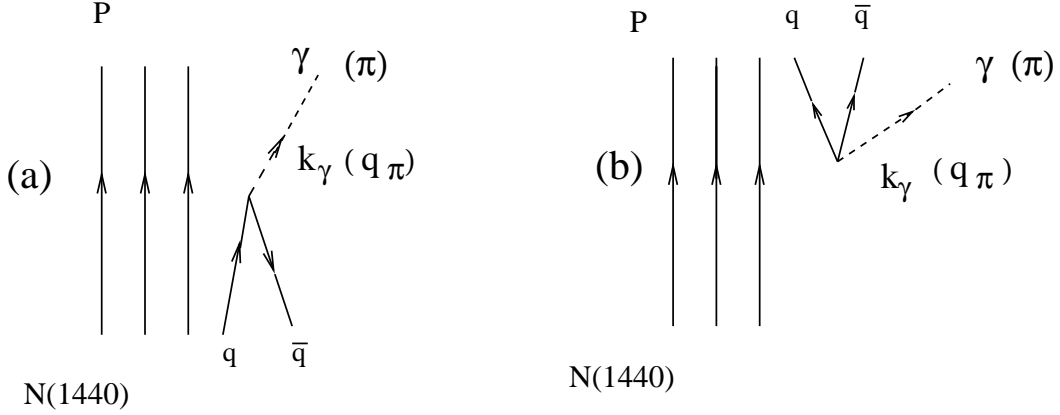


FIG. 2: Direct $q\bar{q} \rightarrow \gamma(\pi)$ annihilation process.

proton with the residual qqq component that is left in the $N(1440)^+$ after the annihilation of a $d\bar{d}$ pair into a photon. It also requires specification of the spatial part of the $N(1440)^+$ wave function. The spin-flavor-color matrix element that is derived from the wave functions of the proton in (2) and $N(1440)^+$ in (9) is:

$$C_{SFC}^{(5q \rightarrow 3q)} = -\frac{2\sqrt{3}}{9}. \quad (22)$$

The following step is the calculation of orbital matrix element of the direct annihilation transition amplitude. This may be cast in the form:

$$C_O^{(5q \rightarrow 3q)}(k_\gamma) = \langle \phi_{000}(\vec{\xi}_1) \phi_{000}(\vec{\xi}_2) [111]_C \mid \delta(\vec{r}_4 - \vec{r}_5) e^{-i\vec{k}_\gamma \cdot \frac{\vec{r}_4 + \vec{r}_5}{2}} \mid \Psi_C(\{\vec{\xi}_i\}) \rangle, \quad (23)$$

where $[111]_C$ denotes the color singlet state of the final proton in the qqq configuration. Note that only the color symmetry configuration of the $qqqq\bar{q}$ component of initial $N(1440)^+$ that is described by C_3 or,

$$\begin{array}{|c|c|} \hline 1 & 4 \\ \hline 2 & \\ \hline 3 & \\ \hline \end{array}, \quad (24)$$

in Eq. (10) leads to a nonzero matrix element, when multiplied with the color singlet of the qqq component of the proton upon annihilation of the 4-th quark with the 5-th antiquark in $N(1440)$. Hence only the term that is multiplied by C_3 in Eq. (10) contributes to the matrix element (23). Consequently the matrix element reduces to:

$$C_O^{(5q \rightarrow 3q)}(k_\gamma) = \left(\frac{\omega_3 \omega_5}{\omega^2}\right)^3 \left\{ \frac{\sqrt{3}}{4} \left[\left(\frac{\omega_5}{\omega}\right)^2 - 1 \right] + \left(\frac{25}{64} X_1 - \frac{3}{16} X_2 - \frac{3}{128} X_1 \frac{k_\gamma^2}{\omega_5^2} \right) \right\} \frac{k_\gamma}{\omega_5} e^{-\frac{3}{20} \frac{k_\gamma^2}{\omega_5^2}}. \quad (25)$$

Here the X_1 and X_2 are defined as the constants $X_1 = \frac{2}{\sqrt{7}} + \frac{6}{5\sqrt{3}}$ and $X_2 = \frac{2}{\sqrt{7}} + \frac{2}{\sqrt{3}}$. In the derivation of (25) the normalization factor mismatch between the qqq and $qqqq\bar{q}$ components that arises from their unequal size parameters ω_3 (qqq) configuration and ω_5 ($qqqq\bar{q}$) has been taken into account by the following factors:

$$\langle \phi_{000}(\vec{\xi}_i) | \varphi_{000}(\vec{\xi}_i) \rangle = \left(\frac{\omega_3 \omega_5}{\omega^2} \right)^{3/2}, \quad \langle \phi_{000}(\vec{\xi}_i) | \varphi_{200}(\vec{\xi}_i) \rangle = \sqrt{\frac{3}{2}} \left(\frac{\omega_3 \omega_5}{\omega^2} \right)^{3/2} \left[\left(\frac{\omega_5}{\omega} \right)^2 - 1 \right]. \quad (26)$$

Here the notation $\omega = \sqrt{(\omega_3^2 + \omega_5^2)/2}$ has been employed.

From (22) and (25) the helicity amplitude for the direct $qqqq\bar{q} \rightarrow qqq\gamma$ transition is obtained as:

$$A_{\frac{1}{2}}^{(53)} = -\frac{2\sqrt{3}}{9} \frac{e}{\sqrt{K_\gamma}} C_O^{(5q \rightarrow 3q)}(k_\gamma). \quad (27)$$

The magnitude of this helicity amplitude contribution depends sensitively on the size parameters ω_3 and ω_5 . With the parameter values above the numerical value $A_{\frac{1}{2}}^{(a)} = -0.035/\sqrt{\text{GeV}}$ for $Q^2 = 0$. This value is somewhat smaller than that given by the conventional qqq model.

Because of the same spin-flavor structure of the wave functions of the proton and $N(1440)$ in their 3-quark and 5-quark configurations, the $qqq \rightarrow qqqq\bar{q}\gamma$ transition described by Fig. 2(b) also contributes to the decays $N(1440)^+ \rightarrow p$, which is different from the the case of $\Delta(1232)$ decay in refs. [9, 10], where the $qqq \rightarrow qqqq\bar{q}\gamma$ transition does not contribute to the resultant decay amplitudes. In the case of the $qqq \rightarrow qqqq\bar{q}\gamma$ transition the spin-flavor-color matrix element is the same as that for the $qqqq\bar{q} \rightarrow qqq\gamma$ transition (22). The orbital matrix element is however different and takes the form:

$$C_O^{(3q \rightarrow 5q)}(k_\gamma) = \langle \psi_C(\{\vec{\xi}_i\}) | \delta(\vec{r}_4 - \vec{r}_5) e^{-i\vec{k}_\gamma \cdot \frac{\vec{r}_4 + \vec{r}_5}{2}} | \frac{1}{\sqrt{2}} (\phi_{000}(\vec{\xi}_1) \phi_{200}(\vec{\xi}_2) + \phi_{200}(\vec{\xi}_1) \phi_{000}(\vec{\xi}_2)) [111]_C \rangle, \quad (28)$$

where $[111]_C$ denotes the color wave function of $N(1440)$ in the qqq configuration. As only the term multiplied by the color wave function C_3 in $\psi_C(\{\vec{\xi}_i\})$ leads to non-zero overlap in Eq. (28), one obtains the result:

$$C_O^{(3q \rightarrow 5q)}(k_\gamma) = \frac{\sqrt{6}}{4} \left(\frac{\omega_3 \omega_5}{\omega^2} \right)^3 \left[\left(\frac{\omega_3}{\omega} \right)^2 - 1 \right] \frac{k_\gamma}{\omega_5} e^{-\frac{3}{20} \frac{k_\gamma^2}{\omega_5^2}}. \quad (29)$$

The helicity amplitude for the direct $qqq \rightarrow qqqq\bar{q}\gamma$ transition is then obtained as:

$$A_{\frac{1}{2}}^{(35)} = -\frac{2\sqrt{3}}{9} \frac{e}{\sqrt{K_\gamma}} C_O^{(3q \rightarrow 5q)}(k_\gamma). \quad (30)$$

With the parameter values above the numerical value $A_{\frac{1}{2}}^{(a)} = 0.0390/\sqrt{\text{GeV}}$ for $Q^2 = 0$. This is large and has the opposite sign to that of $qqqq\bar{q} \rightarrow qq\gamma$ transition, which further reduces the corresponding value in the qqq model.

Combination of the helicity amplitudes in Eqs. (27) and (30) with that of the direct transitions between the 3-quark and 5-quark components in $N(1440)$ and nucleon leads to the net helicity amplitude:

$$A_{\frac{1}{2}} = A_{p3}A_{r3}A_{\frac{1}{2}}^{(3q)} + A_{p5}A_{r5}A_{\frac{1}{2}}^{(5q)} + A_{p3}A_{r5}A_{\frac{1}{2}}^{(53)} + A_{p5}A_{r3}A_{\frac{1}{2}}^{(35)}. \quad (31)$$

Assuming again a 10% and 30% proportion of five quark component in proton and $N(1440)$ and the phase +1 for both five quark components leads at the photon point, $Q^2 = 0$, to the result $A_{\frac{1}{2}} = -0.022/\sqrt{\text{GeV}}$. This value is smaller than that derived in the qqq model by a factor 0.6. Similar numerical results holds even when a proportion of 5-quark component in $N(1440)$ larger than 30% is assumed.

2. $q\bar{q} \rightarrow \gamma$ transitions triggered by quark-quark interactions

Quark-antiquark annihilation transitions may also be triggered by the interactions between the quarks in the baryons. The most obvious such triggering interaction is the confining interaction, which is illustrated diagrammatically in Fig. 3. The role of this mechanism on the calculated strong and electromagnetic decays of the $\Delta(1232)$ has been considered recently in Refs. [9, 10]. The role of confinement driven annihilation on the decays of the $N(1440)^+$ is considered here.

To lowest order in the quark momenta the amplitude for this confinement triggered annihilation mechanism may, in the case of a linear confining interaction, be derived by means of the following replacement in the transition operator for direct annihilation (21):

$$e_i\sigma_{i-} \rightarrow e_i\sigma_{i-}\left(1 - \frac{cr_{ij} - b}{2m}\right). \quad (32)$$

Here c is the string tension, r_{ij} is the distance between the two quarks that interact by the confining interaction and b is a constant, which shifts the zero point of the confinement to negative values ($b > 0$). This replacement applies for both scalar and vector coupled confinement. If the confining interaction is assumed to have the color coupling $\vec{\lambda}_i^C \cdot \vec{\lambda}_j^C$, the string tension c should be the same for all the qq and $q\bar{q}$ pairs in the $qqqq\bar{q}$ system, and half

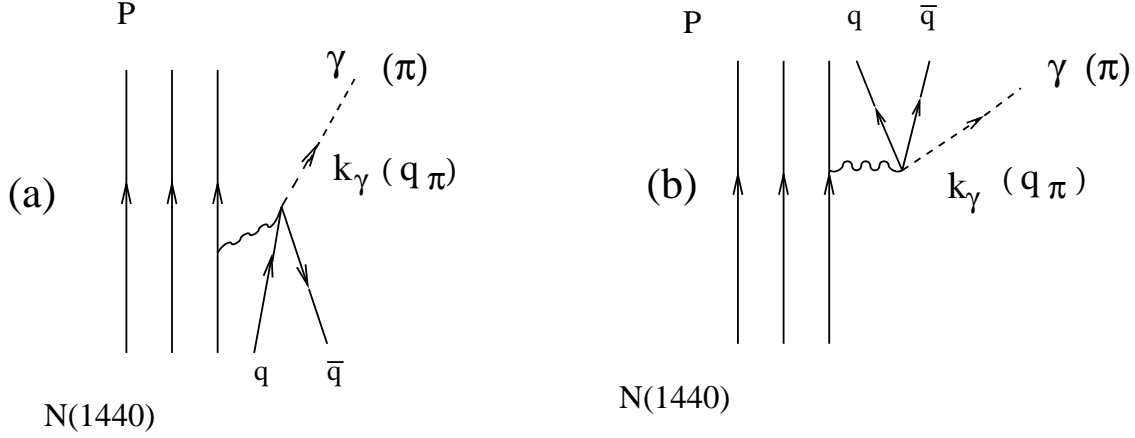


FIG. 3: Confinement induced $q\bar{q} \rightarrow \gamma(\pi)$ annihilation process.

as strong as the corresponding value for quark pairs in three-quark systems. Here we use the value $c = 280$ MeV/fm.

To derive the transition amplitude of the confinement triggered annihilation mechanism one requires the matrix element of the transition operator in orbital space, which, in the case of the $qqqq\bar{q} \rightarrow qq\gamma$ described by Fig. 3(a), is defined as:

$$X_O^{(5q \rightarrow 3q)}(k_\gamma) = \langle \phi_{000}(\vec{\xi}_1) \phi_{000}(\vec{\xi}_2) [111]_C \mid \delta(\vec{r}_4 - \vec{r}_5) e^{-i\vec{k}_\gamma \cdot \frac{\vec{r}_4 + \vec{r}_5}{2}} \left(\frac{cr_{34} - b}{2m} \right) \mid \Psi_C(\{\vec{\xi}_i\}) \rangle. \quad (33)$$

Explicit evaluation leads to the result:

$$X_O^{(5q \rightarrow 3q)}(k_\gamma) = \left(\frac{\omega_3 \omega_5}{\omega^2} \right)^3 \frac{64\sqrt{15}}{375\pi} \frac{c}{m\omega} \frac{k_\gamma}{\omega_5} \left\{ \frac{\sqrt{6}}{2} \left[\left(\frac{\omega_5}{\omega} \right)^2 - 1 \right] K_0(k_\gamma) + \frac{\sqrt{6}}{3} \left(\frac{\omega_5}{\omega} \right)^2 K_1(k_\gamma) + \frac{\sqrt{2}}{2} K_2(k_\gamma) \right\}. \quad (34)$$

Here the functions $K_i(q)$, $i=0, 1, 2$ have been defined as:

$$\begin{aligned} K_0(q) &= \omega_5^5 \int_0^\infty d\xi \xi^4 \frac{j_1(\beta q \xi)}{\beta q \xi} e^{-\alpha^2 \xi^2} k_0(\omega \xi), \\ K_1(q) &= \omega_5^5 \int_0^\infty d\xi \xi^4 \frac{j_1(\beta q \xi)}{\beta q \xi} e^{-\alpha^2 \xi^2} k_1(\omega \xi), \\ K_2(q) &= \omega_5^7 \int_0^\infty d\xi \xi^4 (X_1 \xi^2 - X_2 \frac{3}{2\omega_5^2}) \frac{j_1(\beta q \xi)}{\beta q \xi} e^{-\alpha^2 \xi^2} k_0(\omega \xi). \end{aligned} \quad (35)$$

The constants α and β in this expression are defined as $\alpha = 2\omega_5/\sqrt{5}$ and $\beta = 2\sqrt{3}/5$,

respectively. The functions $k_i(y)$, $i=0, 1$, is defined as:

$$\begin{aligned} k_0(y) &= \int_0^\infty dx x^2 e^{-x^2} \int_{-1}^1 dz \left\{ \sqrt{x^2 - 2\sqrt{2}xyz + 2y^2} - \frac{\sqrt{6}b\omega}{2c} \right\}, \\ k_1(y) &= \int_0^\infty dx x^2 e^{-x^2} \left(x^2 - \frac{3\omega^2}{2\omega_5^2} \right) \int_{-1}^1 dz \left\{ \sqrt{x^2 - 2\sqrt{2}xyz + 2y^2} - \frac{\sqrt{6}b\omega}{2c} \right\}. \end{aligned} \quad (36)$$

The space-flavor-color matrix element of the confinement driven annihilation process is the same as that of the direct annihilation given by (22). Combining with the orbital matrix element in (34), one obtains the following contribution to the helicity amplitudes for $N(1440) \rightarrow N\gamma$ decay:

$$A_{\frac{1}{2}}^{(53c)} = \frac{2\sqrt{3}}{3} \frac{e}{\sqrt{K_\gamma}} X_O^{(5q \rightarrow 3q)}(k_\gamma). \quad (37)$$

Here an overall factor 3 has been inserted for the 3 similar interacting processes of annihilating the antiquark. At $Q^2 = 0$, the numerical value of this contribution is large and positive in the case that $b = 0$: $A_{\frac{1}{2}}^{(53c)} = 0.111/\sqrt{\text{GeV}}$.

The orbital matrix element for the $qqq \rightarrow qq\bar{q}q\bar{q}\gamma$ transition described by Fig. 3(b) is defined as

$$\begin{aligned} X_O^{(3q \rightarrow 5q)}(k_\gamma) &= \langle \psi_C(\{\vec{\xi}_i\}) | \delta(\vec{r}_4 - \vec{r}_5) e^{-i\vec{k}_\gamma \cdot \frac{\vec{r}_4 + \vec{r}_5}{2}} \left(\frac{cr_{34} - b}{2m} \right) | \\ &\quad \frac{1}{\sqrt{2}} (\phi_{000}(\vec{\xi}_1) \phi_{200}(\vec{\xi}_2) + \phi_{200}(\vec{\xi}_1) \phi_{000}(\vec{\xi}_2)) [111]_C \rangle, \end{aligned} \quad (38)$$

which leads explicitly to

$$X_O^{(3q \rightarrow 5q)}(k_\gamma) = \left(\frac{\omega_3 \omega_5}{\omega^2} \right)^3 \frac{128\sqrt{5}}{375\pi} \frac{c}{m\omega} \frac{k_\gamma}{\omega_5} \left\{ \frac{3}{2} \left[\left(\frac{\omega_3}{\omega} \right)^2 - 1 \right] K_0(k_\gamma) + \left(\frac{\omega_3}{\omega} \right)^2 K_1(k_\gamma) \right\}, \quad (39)$$

where

$$\begin{aligned} K_1'(q) &= \omega_5^5 \int_0^\infty d\xi \xi^4 \frac{j_1(\beta q \xi)}{\beta q \xi} e^{-\alpha^2 \xi^2} k_1'(\omega \xi), \\ k_1'(y) &= \int_0^\infty dx x^2 e^{-x^2} \left(x^2 - \frac{3\omega^2}{2\omega_3^2} \right) \int_{-1}^1 dz \left\{ \sqrt{x^2 - 2\sqrt{2}xyz + 2y^2} - \frac{\sqrt{6}b\omega}{2c} \right\}. \end{aligned} \quad (40)$$

Combining with the orbital matrix element, one obtains the following contribution from the $qqq \rightarrow qq\bar{q}q\bar{q}\gamma$ transition to the helicity amplitudes for $N(1440) \rightarrow N\gamma$ decay:

$$A_{\frac{1}{2}}^{(35c)} = \frac{2\sqrt{3}}{3} \frac{e}{\sqrt{K_\gamma}} X_O^{(3q \rightarrow 5q)}(k_\gamma). \quad (41)$$

At $Q^2 = 0$, the numerical value of this contribution is small and negative in the case that $b = 0$: $A_{\frac{1}{2}}^{(35c)} = -0.092/\sqrt{\text{GeV}}$.

The role of the $q\bar{q} \rightarrow \gamma$ on the $N(1440)^+ \rightarrow p\gamma$ decay naturally depends on the phase of the wave functions of the $qqqq\bar{q}$ components of the proton and the $N(1440)$. Combination of all the contributions above to the helicity amplitude with 10% $qqqq\bar{q}$ component in the proton and 30% in $N(1440)$ as above, but assuming a phase -1 for the $qqqq\bar{q}$ component, or $A_{p5} = -0.32$ and $A_{r5} = -0.55$ leads, with $b = 0$, to a calculated value for the helicity amplitude $A_{\frac{1}{2}}$ of -0.056 . This corresponds to an enhancement of the calculated decay width by a factor 2.3 over that, which is derived in the qqq quark model.

Hadron phenomenology suggests that the confining interaction potential should be negative at small distances [11]. That is brought about by taking the value for b to be positive. For instance, when $b = 200$ MeV we get $A_{\frac{1}{2}} = -0.049$ at $Q^2 = 0$. The enhancement of the decay width over that in the qqq model is virtually dominated by the confinement triggered annihilation transitions if $b < 400$ MeV. For larger values of b the contribution from the confinement triggered annihilation amplitude would serve to increase the disagreement between the calculated and the empirical values. To illustrate this the helicity amplitude is plotted in Fig. 4 as a function of Q^2 both for the basic qqq model and when the contributions from elastic $qqqq\bar{q}$ transition (19), the direct annihilation transitions and the confinement triggered annihilation transitions with $b = 0$ and $b = 200$ MeV are taken into account.

The qualitative features of the momentum dependence of the calculated helicity amplitude $A_{1/2}$ in Fig. 4 do not depend very strongly on the presence of the $qqqq\bar{q}$ component, but the momentum behavior does follow the empirical values somewhat better in the presence of those components. The $qqqq\bar{q}$ component brings the value at the photon point closer to the empirical value, and the amplitude has a steeper rise at small values of Q^2 and a change of sign of $A_{\frac{1}{2}}$, also in agreement with the recent empirical values [12]. The magnitude of the calculated helicity amplitude in all considered cases drops towards zero at about 1 GeV². The shape of the calculated helicity amplitude is rather insensitive to the precise value of ω_5 (or to the spatial extent of the $qqqq\bar{q}$ component). Increasing of the numerical value of ω_5 flattens the calculated curve. If the value of ω_5 is increased to 300 MeV the value of $A_{1/2}(0)$ is reduced to -0.034 GeV^{-1/2}, but the shape of the calculated function is affected only marginally.

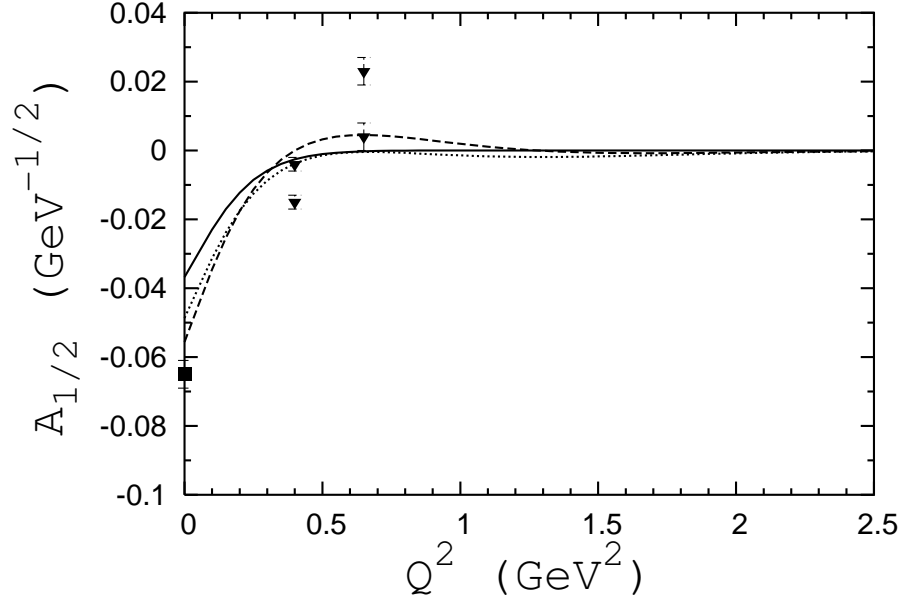


FIG. 4: The helicity amplitude for $N(1440)^+ \rightarrow p\gamma$. Solid line: The result for the conventional qqq model; dashed and dotted lines: the results when the amplitudes for the $qqqq\bar{q} \rightarrow qqqq\bar{q}$ transition and the direct and confinement triggered transitions through quark-antiquark annihilation are combined with the amplitude in the qqq model with $b=0$ and $b=200$ MeV, respectively. Here the proportion of the 5-quark components are assumed to be 10% for the proton and 30% for the $N(1440)$, respectively. The data point at $Q^2 = 0$ is from [7] (square) and the other points are taken from the phenomenological analysis in [12] (triangles).

3. The role of other configurations in the $qqqq\bar{q}$ components

Above the consideration of the $qqqq\bar{q}$ components was restricted to the case, where the 4-quark subsystem has flavor-spin symmetry $[4]_{FS}[22]_F[22]_S$, and which is expected to have the lowest energy of all $qqqq\bar{q}$ configurations. The next to lowest energy $qqqq\bar{q}$ configuration in both the nucleon and the $N(1440)$ is that for which the flavor-spin symmetry is $[4]_{FS}[31]_F[31]_S$ (12) [4]. As this configuration has the same color-space wave function as the already considered configuration $[4]_{FS}[22]_F[22]_S$, no significant change in the calculated results are to be expected by introduction of admixtures with the flavor-spin symmetry $[4]_{FS}[31]_F[31]_S$.

Here only the $qqqq\bar{q} \rightarrow qqq$ and $qqq \rightarrow qqqq\bar{q}$ transitions for the $[4]_{FS}[31]_F[31]_S$ configuration are considered as the contributions from the transitions between five quark components in the $[4]_{FS}[22]_F[22]_S$ and $[4]_{FS}[31]_F[31]_S$ configurations are small. The spin-flavor-color factor, that corresponds to (22) is obtained as,

$$C_{SFC}^{[31]} = -\frac{10\sqrt{6}}{27}, \quad (42)$$

both for the $qqqq\bar{q} \rightarrow qqq$ and $qqq \rightarrow qqqq\bar{q}$ transitions. This is larger than that for $[4]_{FS}[31]_F[31]_S$ (22). The orbital matrix elements of annihilation transition amplitude for the $[4]_{FS}[31]_F[31]_S$ configuration are the same as those of the $[4]_{FS}[22]_F[22]_S$ configuration which are given by Eqs. (23), (28), (33) and (38).

Here both the direct and confinement triggered annihilation transitions $qqqq\bar{q} \rightarrow qqq$ and $qqq \rightarrow qqqq\bar{q}$ in the $[4]_{FS}[31]_F[31]_S$ configuration of proton and $N(1440)^+$ are considered as above. Because the $[4]_{FS}[31]_F[31]_S$ configuration is expected to have a higher energy than the $[4]_{FS}[22]_F[22]_S$ configuration, it should be expected to have a smaller probability than the latter configuration in the five quark component. With the same proportion of five quark component in proton and $N(1440)$ as before, but with 80% of $[4]_{FS}[22]_F[22]_S$ configuration and 20% $[4]_{FS}[31]_F[31]_S$ configuration, the final helicity amplitude is obtained as $A_{\frac{1}{2}} = -0.066/\sqrt{\text{GeV}}$ with $b=200$ MeV at the real photon point. This value falls within the range of the empirical value [7]. In Fig. 5 the calculated helicity amplitudes are shown with both configurations are included. In comparison to the helicity amplitude with only the $[4]_{FS}[22]_F[22]_S$ configuration taken into account the new configuration does not lead to any qualitative change from the previous results, even though it does improve the agreement quantitatively somewhat.

IV. THE STRONG DECAY $N(1440)^+ \rightarrow p\pi^0$

A. The $qqq \rightarrow qqq\pi$ transition

In the "chiral quark" model pions couple directly to constituent quarks. In the non-relativistic approximation the transition operator for $q \rightarrow q\pi^0$ approximation is then:

$$T_\pi = -i\frac{g_A^q}{2f_\pi} \sum_i \tau_z^i \sigma_z^i q_{\pi z}. \quad (43)$$

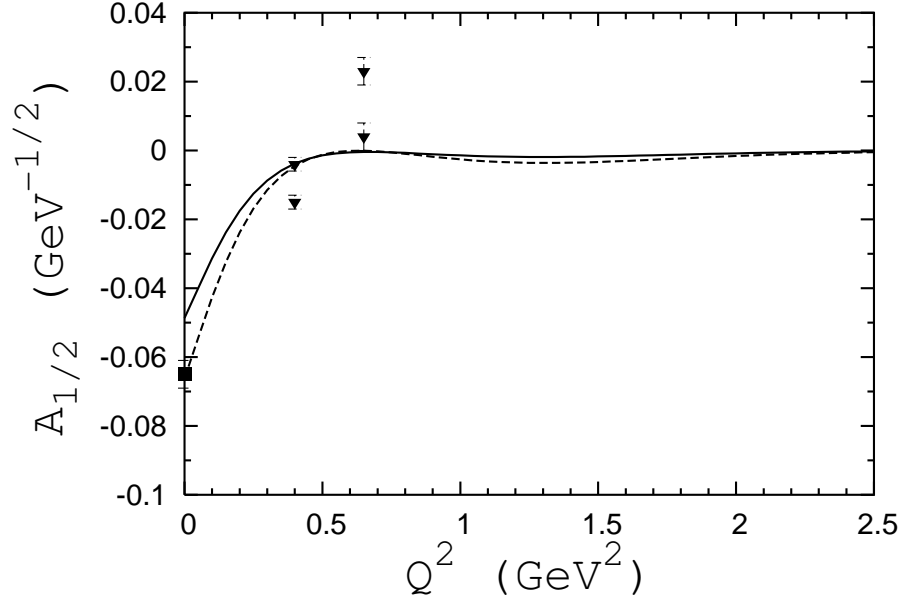


FIG. 5: The helicity amplitude of the decay $N(1440)^+ \rightarrow p\gamma$ as a function of Q^2 . Solid line: only $[4]_{FS}[22]_F[22]_S$ configuration; Dashed line: both $[4]_{FS}[22]_F[22]_S$ and $[4]_{FS}[31]_F[31]_S$ configurations are considered. Here the probability of the $[4]_{FS}[22]_F[22]_S$ configuration in the $qqqq\bar{q}$ component is taken as be 80%. The data point at $Q^2 = 0$ is from [7] (square) and the other points are taken from the phenomenological analysis in [12] (triangles).

Here the sum runs over the quarks, and g_A^q is the axial vector coupling constant of the constituent quarks, f_π the pion decay constant and $q_{\pi z}$ the z-component of the pion momentum.

With the conventional SU(6) qqq wave functions (2) the transition operator for $N(1440)^+(s_z = 1/2) \rightarrow p(s_z = 1/2)\pi^0$, the transition matrix element of the operator (43) in the valence quark model is derived as:

$$T_\pi^{(3q)} = i \frac{5\sqrt{3}}{108} \frac{g_A^q}{f_\pi} \frac{\vec{q}_\pi^2}{\omega_3^2} e^{-\vec{q}_\pi^2/6\omega_3^2} q_{\pi z}. \quad (44)$$

Here the spatial extent of the qqq component of the baryon in the harmonic oscillator model has been taken into account. For the case of $N(1440)^+ \rightarrow p\pi^0$ the pion momentum is $q_\pi = 397$ MeV.

The amplitude (44) leads to values for the decay width for $N(1440) \rightarrow N\pi$ that corresponds only to about $\sim 1/6$ of the empirical value [13]. This deficiency is a generic feature

of the qqq quark model [1, 2].

B. The $qqqq\bar{q} \rightarrow qqqq\bar{q}\pi$ transition

The transition operator (43) leads to the following contribution to the pion decay amplitude that connects the $qqqq\bar{q}$ components, in the $[4]_{FS}[22]_F[22]_S$ mixed symmetry flavor-spin configuration, of the $N(1440)^+$ (9) and the proton (5):

$$T_{\pi}^{(5q)} = -i \frac{\sqrt{6}}{180} \frac{g_A^q}{f_{\pi}} \frac{\vec{q}_{\pi}^2}{\omega_{\pi}^2} e^{-\vec{q}_{\pi}^2/5\omega_{\pi}^2} q_{\pi z}. \quad (45)$$

Note that, similarly to the case of the electromagnetic decay of the $N(1440)^+$, the decay amplitude in (45) only arises from the $\bar{q} \rightarrow \bar{q}\pi^0$ transition, while the quark transitions $q \rightarrow q\pi^0$ gives no contribution in the configuration with [22] spin symmetry.

The transition amplitude for pion decay between the $qqqq\bar{q}$ components of $N(1440)$ and the proton has the opposite sign to that given by the qqq model. These $qqqq\bar{q}$ components would therefore decrease the calculated final decay width when the effect of these mechanisms are combined if the cross term matrix elements are not taken into account. With a 10% proportion of 5-quark component in proton and 30% in $N(1440)$, these terms would imply a reduction factor of 0.6 of the calculated decay width.

C. The $q\bar{q} \rightarrow \pi$ transition

1. $q\bar{q} \rightarrow \pi$ transition without quark-quark interactions

The simplest $qqqq\bar{q} \rightarrow qqq\pi$ and $qqq \rightarrow qqqq\bar{q}\pi$ decay mechanisms that may contribute to the strong decay of the $N(1440)$ are the $q\bar{q} \rightarrow \pi$ pair annihilation process which is shown in Fig. 2. The corresponding amplitude is

$$T_{\pi a} = -i \frac{mg_A^q}{f_{\pi}} \tau_z, \quad (46)$$

where τ_z represents the z-component of the isospin of the annihilating quark.

With the transition operator (46) the spin-flavor-color matrix element is derived from the wave functions of the proton in (2) and $N(1440)^+$ in (9) as

$$C_{SFC}^{(5q \rightarrow 3q)} = \frac{\sqrt{3}}{6}, \quad (47)$$

in the case where the $qqqq\bar{q}$ component has flavor- spin symmetry $[4]_{FS}[22]_F[22]_S$. Multiplication with the orbital matrix element (23) the final amplitude for the $N(1440)^+ \rightarrow p\pi^0$ decay in the $qqqq\bar{q} \rightarrow qq\pi$ mode is obtained as:

$$T_\pi^{(53)} = -i \frac{2\sqrt{3}}{3} \frac{mg_A^q}{f_\pi} C_O^{(5q \rightarrow 3q)}(q_\pi). \quad (48)$$

Here an overall factor 4 has been introduced to account for the numbers of quarks that may annihilate the antiquark.

Consider then the $qqq \rightarrow qqqq\bar{q}\pi$ transition described by Fig. 2(b). The amplitude for the direct $qqq \rightarrow qqqq\bar{q}\pi$ transition is obtained as:

$$T_\pi^{(35)} = -i \frac{2\sqrt{3}}{3} \frac{mg_A^q}{f_\pi} C_O^{(3q \rightarrow 5q)}(q_\pi). \quad (49)$$

The matrix element in spin-flavor-color space is the same in this case as in the case of the $qqqq\bar{q} \rightarrow qq\pi$ transition in Eq. (47). The orbital matrix is given in Eq. (29).

As the amplitudes in (48) and (49) have the opposite sign to the one in the qqq model, they would lead to an enhancement of the calculated decay width with the phase -1 in the wave functions of proton and $N(1440)$ as above. Again, with the assumption of 10% and 30% $qqqq\bar{q}$ components in proton and the Roper resonance respectively as above, these annihilation and creation contributions lead to an enhancement factor of 1.3 for the calculated decay width for $N(1440) \rightarrow p\pi^0$, when the amplitude in (48) is combined with that of the transitions between the qqq and $qqqq\bar{q}$ components. By itself this enhancement is not large enough to compensate for the underprediction of the empirical width in the quark model.

2. $q\bar{q} \rightarrow \pi$ transition triggered by quark-quark interactions

The strong decay of $N(1440)$ through quark-antiquark annihilation can also be triggered by the interactions between the quarks in the baryons, as illustrated by Fig. 3. With the assumption of a linear scalar coupled confining interaction between the quarks the amplitude for the confinement triggered annihilation mechanism in the pion decay of $N(1440)$, may to lowest order in the quark momenta be derived by making the following replacement in the transition operator for direct annihilation (21):

$$m \rightarrow m + \frac{1}{2}(cr_{ij} - b). \quad (50)$$

With the spin-flavor-color matrix element (47) and the orbital matrix elements (33) and (38) the amplitudes for quark-antiquark annihilation triggered by the confining interaction is found to be

$$T_{\pi}^{(53c)} = -i2\sqrt{3}\frac{mg_A^q}{f_{\pi}} X_O^{(5q\rightarrow 3q)}(q_{\pi}), \quad (51)$$

for the $qqqq\bar{q} \rightarrow qq\pi$ transition and

$$T_{\pi}^{(35c)} = -i2\sqrt{3}\frac{mg_A^q}{f_{\pi}} X_O^{(3q\rightarrow 5q)}(q_{\pi}), \quad (52)$$

for the $qqq \rightarrow qqqq\bar{q}\pi$ transition. Here an overall factor 12 has been introduced to account for the number of the similar processes of the confinement triggered annihilation of the quarks with the antiquark.

The effect of the amplitude of the confinement triggered annihilation transition (51) on the final strong decay width of $N(1440)$ is very sensitive to the value of the shift parameter b in the linear confining interaction. With the value $b = 200$ MeV, used above for the helicity amplitude for electromagnetic decay, the the confinement triggered annihilation transition amplitude leads to an enhancement of the calculated decay width of 6.1, when it is combined with the amplitudes of the transition between the $qqqq\bar{q}$ components and the direct annihilation transitions. With $b = 0$ the enhancement factor is 7.9. This enhancement is of the right order of magnitude to compensate for the underprediction of the decay width in the conventional qqq quark model (44).

V. DISCUSSION

The result above shows that by explicit introduction of a sizable, $\sim 30\%$, $qqqq\bar{q}$ component in the low lying broad $N(1440)$ resonance and a smaller such component in the proton, it becomes possible to describe the helicity amplitude $A_{\frac{1}{2}}$ for $N(1440) \rightarrow N\gamma$ decay and the empirical width for $N(1440) \rightarrow N\pi$ decay with the same set of quark model parameters. Without such extension of the quark model it is difficult to understand the observed structure and the large decay width of the low lying $N(1440)$ resonance. The fact that this resonance may be described as a vibrational mode of the nucleon - i.e. as a collective state - in the Skyrme model also suggests that it should contain considerable multiquark components [3]. This may also be indirectly inferred from the fact that this resonance may be generated dynamically in phenomenological hadronic coupled channel models [14].

With properly chosen size parameters of nucleon and $N(1440)$, the extension of the quark model to include $qqqq\bar{q}$ components considered here leads to a transition form factor for the $N(1440)$ resonance, that has the same strong momentum dependence that is suggested by the phenomenological analysis of recent data in ref.[12]. This feature requires that the wave function of the resonance be more extended spatially than that of the nucleon.

Besides the $qqqq\bar{q}$ configuration with the isospin-spin symmetry $[4]_{FS}[22]_F[22]_S$, which is expected to have the lowest energy because of the spin dependence of the hyperfine interaction between the quarks, other configurations such as $[4]_{FS}[31]_F[31]_S$ may also contribute to the final transition form factor quantitatively. The $[4]_{FS}[22]_F[22]_S$ symmetry is rather restrictive, in that it only allows the antiquark in the proton and the $N(1440)$ to be a \bar{d} antiquark. The $qqqq\bar{q}$ configuration with $[4]_{FS}[31]_F[31]_S$ flavor-spin symmetry is expected to have the next lowest energy and would also allow \bar{u} antiquarks to be admixed into these baryons. This symmetry configuration was also studied numerically above, but was nevertheless found to bring no qualitative change to the numerical results. The fact that empirically the \bar{d}/\bar{u} ratio is considerably larger than 1 in the proton does nevertheless indicate that to a first approximation, it may be sufficient to consider only the configuration $[4]_{FS}[22]_F[22]_S$ [15].

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